1 $\exists x(Fx \wedge Gx), \forall x(Fx \rightarrow Hx) \vdash \exists x(Gx \wedge Hx)$

1	$\exists x(Fx \wedge Gx)$	
2	$\forall x(Fx \rightarrow Hx)$	
3	$Fu \wedge Gu$	
4	$Fu \rightarrow Hu$	$\forall$B:2
5	Fu	$\wedge$B:3
6	Hu	$\rightarrow$B:4,5
7	Gu	$\wedge$B:3
8	$Gu \wedge Hu$	$\wedge$E:6,7
9	$\exists x(Gx \wedge Hx)$	$\exists$E:8
10	$\exists x(Gx \wedge Hx)$	$\exists$B:1,3–9

2 $\exists x \forall y Lxy \vdash \forall x \exists y Lyx$

1	$\exists x \forall y Lxy$	
2	$\forall y Luy$	
3	Luv	$\forall$B:2
4	$\exists y Lyv$	$\exists$E:3
5	$\forall x \exists y Lyx$	$\forall$E:4
6	$\forall x \exists y Lyx$	$\exists$B:1,2–5

Alternativ kann man die $\forall$E auch später ausführen:

1	$\exists x \forall y Lxy$	
2	$\forall y Luy$	
3	Luv	$\forall$B:2
4	$\exists y Lyv$	$\exists$E:3
5	$\exists y Lyv$	$\exists$B:1,2–4
6	$\forall x \exists y Lyx$	$\forall$E:5

3 $\exists x \forall y Lxy \vdash \exists x Lxx$

1	$\exists x \forall y Lxy$	
2	$\quad \forall y Luy$	
3	$\quad Luu$	∀B:2
4	$\quad \exists x Lxx$	∃E:3
5	$\exists x Lxx$	∃B:1,2–4

4 $\exists x(Fx \wedge Gx) \vdash \neg \forall x(Fx \rightarrow \neg Gx)$

1	$\exists x(Fx \wedge Gx)$	
2	$\quad Fu \wedge Gu$	
3	$\quad\quad \forall x(Fx \rightarrow \neg Gx)$	
4	$\quad\quad Fu \rightarrow \neg Gu$	∀B:3
5	$\quad\quad Fu$	∧B:2
6	$\quad\quad \neg Gu$	→B:4,5
7	$\quad\quad Gu$	∧B:2
8	$\quad\quad Gu \wedge \neg Gu$	∧E:6,7
9	$\quad \neg \forall x(Fx \rightarrow \neg Gx)$	¬E:3–8
10	$\neg \forall x(Fx \rightarrow \neg Gx)$	∃B:1,2–9